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On the optimization of trading on the grain market with the grain storage cost

O optymalizacji handlu na rynku zbożowym z kosztem przechowywania ziarna

Abstract: In the article the influence of the increase in the grain storage cost on the effect of the optimization of trading on the grain market is considered. The result of the execution of the strategy of the grain purchase can be characterized for example by the sum of the expected value of the amount of money paid for the purchase of the grain and the cost of the storage of this grain. The process of the market price of the grain can be determined for example by the forecast of the grain price in the future. From the exemplary numerical calculations it follows that the storage may have a significant impact on the result of the optimization of the trading strategy on the grain market.

Key words: grain price process, grain storage cost

JEL codes: C6, Q14

Synopsis. W artykule rozważany jest wpływ wzrostu kosztu przechowywania ziarna na wynik optymalizacji strategii handlowej na rynku zbożowym. Wynik realizacji strategii kupna zboża może być charakteryzowany na przykład przez sumę wartości oczekiwanej kwoty wydanej na zakup zboża i kosztu jego przechowywania. Proces ceny ziarna może być określony na przykład przez prognozę przyszłej ceny zboża. Z przykładowych obliczeń numerycznych wynika, że magazynowanie ziarna może mieć istotny wpływ na wynik optymalizacji strategii handlu na rynku zbożowym.

Słowa kluczowe: proces ceny zboża, koszt magazynowania ziarna

Introduction

The cultivation of the cereals is one of the main sectors of the agricultural production in Poland. The cereal grain is used for example in the production of flour. The cereals are important in the food industry. The price dynamics of the grain may significantly affect the level of profitability of the agricultural production. The form of the grain process is important in planning a strategy of purchasing the grain. The typical feature of the grain price process is

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the uncertainty about its future realization. The dynamics of the grain price can be modelled by a stochastic process with the parameters determined on the basis of the grain market analysis. The grain market is characterized by the volatility of the grain prices. The variability of the grain prices was studied for example in Borkowski and Krawiec [2010], Hamulczuk and Klimkowski [2011], Krawiec [2011] and Olszańska [2011]. The important element characterizing the dynamics of the grain price is the grain price trend. Forecasting a trend in the process of the future grain price may be important in optimizing a buying strategy on the grain market. In the period between the consecutive harvests, there may be the upward trend in the process of the grain price due to declining the amount of the grain on the market. The important factor that may generate a trend in the grain price is weather. The occurrence of the shortage of water may be the factor implying the increase in the price of the grain. The grain prices may depend on the international grain trade [Ginter and Szarek 2010] and the grain prices may be significantly influenced by the legal regulations for example the legal restrictions on the import and export of the grain. The forecasting of the prices on the grain market was analyzed for example in Jodź and Mruklik [2014], Tłuczak [2010] and Tłuczak and Szewczyk [2010]. The forecasted, sufficiently high, increase in the expected price of the grain is the premise of storing the grain. Agriculture is a branch of the economy where the importance of the storage may increase with the growth of the farm [Rokicki and Wicki, 2010]. The grain storage is the activity where there is the risk of the grain quality deterioration and therefore it is important to control the moisture and the temperature of the stored grain [Tukiendorf et al. 2007, Kaleta and Górnicki 2008]. Reducing the risk of a deterioration of the quality of the grain during the grain storage may generate a significant cost. Logistic in agriculture was analyzed for example in [Klepacki 2016].

The aim of this article is to determine the grain purchase strategy which minimizes the expected amount of money spent on the purchase and the storage of the grain in a finite interval and to analyze, on the basis of the exemplary numerical calculations, the impact of the grain storage on the result of the optimization of the purchase of the grain.

The model of the grain market and the description of the strategy of the purchase and the utilization of the grain

Let C_t symbolize the market price per tonne of the grain at time t. The process of the grain market price is given as follows:

$$C_t = C_0\left(1 + \mu t + \sigma Y_t\right) \text{ for } t \in [0, \mathrm{T}), \tag{1}$$

where $\sigma > 0$, μ symbolizes the speed of the change in the average market price of the grain in the interval $[0, \mathrm{T})$, and $\{Y_t : t \in [0, T]\}$ is a stochastic process such that

$$E(Y_t) = 0 \tag{2}$$

and $C_t > 0$ for each $t \in [0, T)$.

The exemplary picture of the trajectory of the market grain price with the expected market grain price as the function of time is shown in Figure 1.

Figure 1. The exemplary trajectory of the grain market price and the exemplary graph of the expected grain market price as the funcion of time
Rysunek 1. Przykładowa trajektoria rynkowej ceny ziarna i przykładowy wykres oczekiwanej ceny ziarna jako funkcji czasu
Source: own elaboration.

The strategy φ of the purchase of the grain in the interval $[0,T)$ can be characterized by the process X^{φ} such that $X^{\varphi}(t)$ denotes the amount of the grain purchased by the market participant in the interval $[0,t)$. The velocity at time t of utilizing the grain purchased in the interval $[0,T)$ is denoted by $v(t)$. Let $Z^{\varphi}(t)$ symbolize the amount of the grain possessed by the market participant at time t by the application of the strategy φ. The value of $Z^{\varphi}(t)$ is given as follows:

$$Z^{\phi}(t) = X^{\phi}(t) - \int_0^t v(s)ds. \tag{3}$$

The strategy φ is such that

$$X^{\phi}(t) \geq \int_0^t v(t)dt \text{ dla } 0 \leq t < T. \tag{4}$$

The condition (4) implies that the amount of the grain purchased by executing the strategy φ is sufficient to utilize this grain with the speed $v(t)$ at the moment t for $0 < t < T$.

Let $EA(\varphi)$ symbolize the expected amount of money spent on the purchase of the grain in the interval $[0,T)$ by the application of the strategy φ.

The cost of the grain storage can increase in the course of time. It is assumed that the expected cost $c_t^{\Delta t}$ of storing a tons of the grain in the interval $[t, t+\Delta t)$ is given as follows:

$$c_t^{\Delta t} = \int_t^{t+\Delta t} (k + \beta s)ds, \tag{5}$$

where $k \geq 0$ and $\beta \geq 0$. The parameter β describes the speed of the increase of the grain storage cost. By (5) it is obtained that

$$c_t^{\Delta t} = k\Delta t + \beta t\Delta t + \frac{\beta}{2}(\Delta t)^2. \tag{6}$$

Consider the moments $\frac{i}{n}T$ for $i = 0,...,n-1$ from the interval $[0,T)$. Denote by c_i the cost of storing a tonne of the grain in the interval $\left[\frac{i}{n}T, \frac{i+1}{n}T\right)$ for $i = 0,...,n-1$. By (6) the value of c_i is given as follows:

$$c_i = \frac{k}{n}T + \beta i\left(\frac{T}{n}\right)^2 + \frac{\beta}{2}\left(\frac{T}{n}\right)^2 \text{ for } i = 0,...,n-1. \tag{7}$$

If the grain storage costs are known for two separate intervals contained in the interval $[0,T)$ then by (7) the parameters k and β can be determined by solving the following system of equations:

$$\begin{cases} c_i = \frac{k}{n}T + \beta i\left(\frac{T}{n}\right)^2 + \frac{\beta}{2}\left(\frac{T}{n}\right)^2 \\ c_j = \frac{k}{n}T + \beta j\left(\frac{T}{n}\right)^2 + \frac{\beta}{2}\left(\frac{T}{n}\right)^2 \end{cases} \text{ for } i \neq j. \tag{8}$$

By (8) the values of the parameters β and k are given as follows:

$$\beta = \frac{c_i - c_j}{i-j}\left(\frac{n}{T}\right)^2, \tag{9}$$

$$k = \frac{c_j(2i+1) - c_i(2j+1)}{2(i-j)}\frac{n}{T}. \tag{10}$$

Consider the example such that the monthly cost of storing one tons of the grain in January in some year is 120 PLN and the monthly cost of storing one tons of the grain in December of this year is 180 PLN. Moreover, in this example, $T = 1$, $n = 12$, $i = 0$ and $j = 11$. Thus, $c_0 = 120$ and $c_{11} = 180$. Consequently, by (9) and (10) it is obtained that $\beta = 392{,}73$ and $k = 1439{,}92$. Figure 2 shows the values of the monthly costs of storing one tons of the grain in this example.

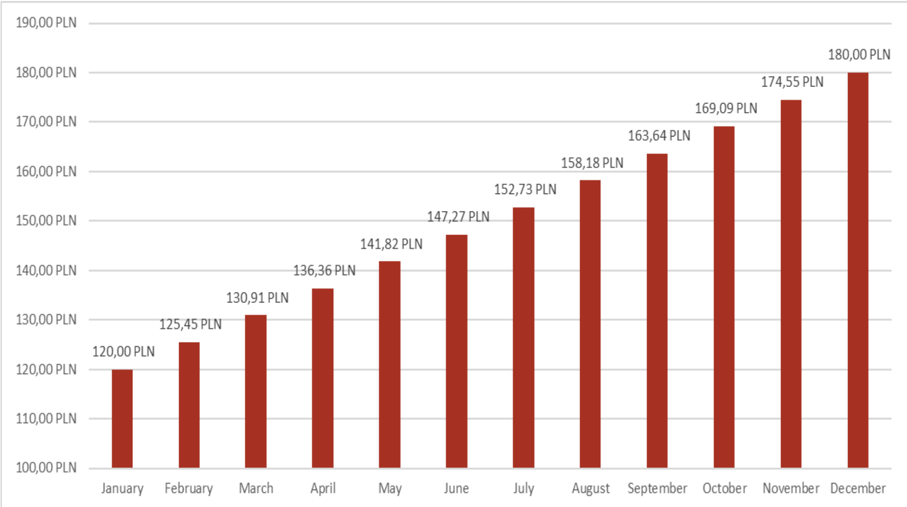

Figure 2. The graph of the exemplary monthly costs of storing one tonne of the grain
Rysunek 1. Wykres przykładowych miesięcznych kosztów przechowywania tony ziarna
Source: own elaboration.

Let $SC(\varphi)$ symbolize the amount of money spent on storing the grain purchased in the interval $[0,T)$ by using the strategy φ. By (5) the value of $SC(\varphi)$ is given by the following formula:

$$SC(\phi)=\int_0^T (k+\beta s) Z^{\phi}(s)ds. \tag{11}$$

Let $EASC(\varphi)$ denote the sum of the expected amount of money spent for the purchase of the grain in the interval $[0,T)$ and the cost of the storage of this grain. Thus,

$$EASC(\varphi)=EA(\varphi)+SC(\varphi).$$

Optimization of the strategy of purchasing the grain

The grain market participant (for example the flour producer) is considered who wants to minimize the sum of the expected amount spent on the purchase of the grain in the interval $[0,T)$ and the amount spent on storing of this grain in the case where the utilization of the grain purchased in the interval $[0,T)$ by the market participant is given as follows:

$$\int_0^t v(s)ds=\eta t \text{ for } t\in[0,T), \text{ where } \eta>0. \tag{12}$$

The condition (12) implies that the market participant utilizes the grain purchased in the interval $[0,T)$ with the constant velocity η in this interval.

Let θ denote the moment from the interval $[0,T)$ and let θ^* symbolize the moment of the grain purchase such that the value of the sum of the expected amount of money spent on the purchase of the grain utilized at time θ and the cost of storing of this grain is minimized over the interval $[0,\theta]$. Let f_θ denote the function of the variable t from the interval $[0,\theta]$ such that the value of $f_\theta(t)$ is the sum of the expected amount of money spent at the moment t on the purchase of one tonne of the grain which is utilized at time θ and the cost of storage of this grain. By (1), (2) and (6) the following formula for the value of $f_\theta(t)$ is obtained:

$$f_\theta(t)=C_0(1+\mu t)+k(\theta-t)+\beta t(\theta-t)+\frac{\beta}{2}(\theta-t)^2 \text{ for } t\in[0,\theta]\,. \tag{13}$$

By the inequality $\beta\geq 0$ and (13) the following formula for θ^* is obtained:

$$\theta^*=\begin{cases}0 \text{ if } \beta\theta\leq 2(\mu C_0-k)\\ \theta \text{ if } \beta\theta>2(\mu C_0-k)\end{cases}. \tag{14}$$

Let φ^* denote the strategy of the grain purchase in the interval $[0,T)$ which minimizes, over the set of trading strategies satisfying (4), the sum of the expected amount spent on the purchase of the grain in the interval $[0,T)$ and the amount spent on storing of this grain in the case where the utilization of the grain purchased in the interval $[0,T)$ by the market participant is given by (12).

Let ψ denote the moment defined by the following equality:

$$\psi=\begin{cases}0 \text{ if } \mu C_0\leq k\\ T \text{ if } \beta=0 \text{ and } \mu C_0>k\\ \min\left\{\frac{2(\mu C_0-k)}{\beta},T\right\} \text{ if } \beta>0 \text{ and } \mu C_0>k\end{cases}. \tag{15}$$

By (4), (12), (14) and (15) it is obtained that the process X^{φ^*} is given as follows:

$$X^{\varphi^*}(t)=\begin{cases}\eta\psi \text{ if } 0\leq t<\psi\\ \eta t \text{ if } \psi\leq t<T\end{cases}, \tag{16}$$

By (3), (12) and (16) it follows that Z^{φ^*} is given by the following formula:

$$Z^{\varphi^*}(t)=\begin{cases}\eta(\psi-t) \text{ if } 0\leq t<\psi\\ 0 \text{ if } \psi\leq t<T\end{cases}. \tag{17}$$

By (1), (17) and (16) it follows that:

$$EA(\varphi^*)=\eta C_0\left(T+\frac{1}{2}\mu(T-\psi)(T+\psi)\right). \tag{18}$$

By (5) and (17) the value of $SC(\varphi^*)$ is given as follows:

$$SC(\varphi^*)=\eta\left(\frac{1}{2}k\psi^2+\frac{1}{6}\beta\psi^3\right). \tag{19}$$

The numerical example

In this section, in the numerical computations, it is assumed that the initial price per tons of the grain equals 1250, $T=0{,}5$ and $\eta=1$. The equality $\eta=1$ implies that the grain market participant in the interval $[0,T)$ utilizes 500 tons of the grain. The results of the calculations of the values of the amount of the sum of the expected amount of money spent on the purchase of the grain in the interval $[0,T)$ and the amount spent on storing of this grain while executing the strategy φ^* for 140 pairs of the values of the parameters β, θ and $k=120$ are shown in Table 1.

Table 1. The exemplary values of $EASC(\varphi^*)$ for $k=120$
Tabela 1. Przykładowe wartości $EASC(\varphi^*)$ dla $k=120$

		μ 0.20	0.40	0.60	0.80	1.00	1.20	1.40
β	**0**	640.00	640.00	640.00	640.00	640.00	640.00	640.00
	200	644.17	644.17	644.17	644.17	644.17	644.17	644.17
	400	648.33	648.33	648.33	648.33	648.33	648.33	648.33
	600	652.18	652.50	652.50	652.50	652.50	652.50	652.50
	800	653.96	656.67	656.67	656.67	656.67	656.67	656.67
	1000	654.79	660.83	660.83	660.83	660.83	660.83	660.83
	1200	655.23	665.00	665.00	665.00	665.00	665.00	665.00
	1400	655.50	669.17	669.17	669.17	669.17	669.17	669.17
	1600	655.68	673.21	673.33	673.33	673.33	673.33	673.33
	1800	655.80	676.21	677.50	677.50	677.50	677.50	677.50
	2000	655.88	678.35	681.67	681.67	681.67	681.67	681.67
	2200	655.95	679.94	685.83	685.83	685.83	685.83	685.83
	2400	656.00	681.15	690.00	690.00	690.00	690.00	690.00
	2600	656.03	682.09	694.09	694.17	694.17	694.17	694.17

cont. Table 1

β	**2800**	656.06		682.83	697.49	698.33	698.33	698.33	698.33
	3000	656.09		683.44	700.23	702.50	702.50	702.50	702.50
	3200	656.11		683.93	702.47	706.67	706.67	706.67	706.67
	3400	656.12		684.34	704.33	710.83	710.83	710.83	710.83
	3600	656.14		684.68	705.89	714.94	715.00	715.00	715.00
	3800	656.15		684.97	707.21	718.54	719.17	719.17	719.17

Source: own elaboration.

The graph of the sum of the expected amount of money spent on the purchase of the grain in the interval $[0,T)$ and the amount spent on storing of this grain while executing the strategy φ^* for $k=120$ is shown in Figure 3.

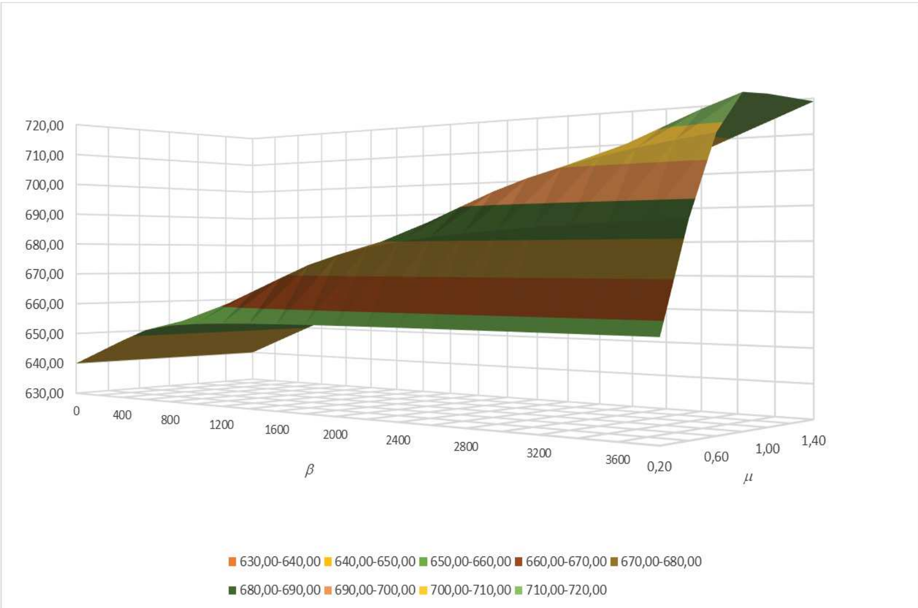

Figure 3. The graph of $EASC(\varphi^*)$ as the function of β and μ for $k=120$
Rysunek 3. Wykres $EASC(\varphi^*)$ jako funkcji β i μ dla $k=120$
Źródło: own elaboration.

From Table 1 it can be seen that value of the amount of the sum of the expected amount of money spent on the purchase of the grain in the interval $[0,T)$ and the amount spent on storing of this grain while executing the strategy φ^* can significantly increase with the increase of the parameter β and the increase in the value of the trend in the grain price can induce the increase of $EASC(\varphi^*)$.

The results of the calculations of the values of the amount of the sum of the expected amount of money spent on the purchase of the grain in the interval $[0,T)$ and the amount spent on storing of this grain while executing the strategy φ^* for 140 pairs of the values of the parameters k and θ, $\beta=1000$ are shown in Table 2.

Table 2. The exemplary values of $EASC(\varphi^*)$ for $\beta=1000$
Tabela 2. Przykładowe wartości $EASC(\varphi^*)$ dla $\beta=1000$

		μ						
		0.20	**0.40**	**0.60**	**0.80**	**1.00**	**1.20**	**1.40**
	0	645.83	645.83	645.83	645.83	645.83	645.83	645.83
	20	648.14	648.33	648.33	648.33	648.33	648.33	648.33
	40	650.08	650.83	650.83	650.83	650.83	650.83	650.83
	60	651.68	653.33	653.33	653.33	653.33	653.33	653.33
	80	652.97	655.83	655.83	655.83	655.83	655.83	655.83
	100	654.00	658.33	658.33	658.33	658.33	658.33	658.33
	120	654.79	660.83	660.83	660.83	660.83	660.83	660.83
	140	655.36	663.33	663.33	663.33	663.33	663.33	663.33
	160	655.76	665.83	665.83	665.83	665.83	665.83	665.83
k	**180**	656.02	668.33	668.33	668.33	668.33	668.33	668.33
	200	656.17	670.83	670.83	670.83	670.83	670.83	670.83
	220	656.23	673.33	673.33	673.33	673.33	673.33	673.33
	240	656.25	675.83	675.83	675.83	675.83	675.83	675.83
	260	656.25	678.28	678.33	678.33	678.33	678.33	678.33
	280	656.25	680.40	680.83	680.83	680.83	680.83	680.83
	300	656.25	682.17	683.33	683.33	683.33	683.33	683.33
	320	656.25	683.61	685.83	685.83	685.83	685.83	685.83
	340	656.25	684.77	688.33	688.33	688.33	688.33	688.33
	360	656.25	685.67	690.83	690.83	690.83	690.83	690.83
	380	656.25	686.35	693.33	693.33	693.33	693.33	693.33

Source: own elaboration.

From Table 2 it follows that value of the amount of the sum of the expected amount of money spent on the purchase of the grain in the interval $[0,T)$ and the amount spent on storing of this grain while executing the strategy φ^* can significantly increase with the increase of the parameter k.

The choice of the values of the parameters of the model used in the numerical example seems to be not very far from the empirical observations of the grain market and is one of the reasonable choices for the exemplary calculations.

Summary

In the article the method the trading strategy φ^* which minimizes the sum of the expected amount of money spent on the purchase the storage of the grain in a finite interval is considered. It is shown how the optimal strategy φ^* depends on the trend in the grain market price and the parameters β and k characterizing the grain storage cost. From the numerical computations included in the article it can be concluded that the grain storage may significantly impact the result of the optimization of the purchase of the grain.

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